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A Concise Study and Analysis on Service Cost and Waiting Cost of the Multi Server Queuing Model

S Vijay Prasad[1], Ranadheer Donthi[2]

[1]Department of Mathematics, Koneru Lakshmaih Education Foundation, Vaddeswaram, Guntur Dist., AP, India-522502

[2]Department of Mathematics, St. Martin's Engineering College, Dhulapally, Kompally, Hyderabad, Telangana, India, 500100

[1]indorevijay@gmail.com

Abstract

Queuing models deals with waiting lines or queues which usually appear at the service providers. In the queuing theory the service cost and waiting cost are conflict to each other and it is very important to balance them. This research article explores some results regarding the analysis of the service and waiting cost of the multi server queuing model and probable number of customers in the queue and in the system which are obtained by using TORA software and interpreted graphically.

Key Words: *M/M/S queuing model, waiting cost, Service cost, Servers, Consumers and Performance measures*.

1. Introduction

The lengthy queue leads loss to a consumer as well as service provider. The service provider with low standards and least expenditure causes the danger of high dissatisfaction in the minds of consumers and there will be damage in the growth of the future business. The stumpy efficiency of the system causes the more number of the people appears in the waiting line. In order to reduce the number of people from the waiting lines the only and simple technique is to increase servers and the efficiency of the service capacity of the system. Adding the more number of servers and increasing the efficiency of the system means additional cost to the service providers. On the other hand the customers are waiting for long time in the queue is generates dissatisfaction in the customer mind which is cost to the customers based on their busy schedule. Therefore these two costs (service cost and waiting cost) are very important to balance in the queuing system. The service cost means the cost of the operating service facility which involves equipment, labour and materials and the waiting cost mean the waiting time of the customers which they spend in the long waiting line and their resources. The waiting cost may be varies from person to person based on their social status and busyness which is highly difficult to determine. In contrast if the service provider offers redundant service to the customers than there is a chance of getting dissatisfaction of service cost. The waiting cost and the service cost are the basic costs which play an important role in running a system without any disturbance. These two costs are conflict to each other because if the service cost is increased which is extra cost to the service providers, or if not which is extra burden the customers. Therefore these two costs must be optimized to satisfy the service provider and the waiting customers. The optimization of the queuing system is to minimize the total cost which is of the service cost and waiting cost. In 2018, Vijay Prasad et. al., in their research article found the required number of servers and expected number of customers in the system of multi server queuing model by using LPP-graphical method. In 2019, Vijay Prasad et. al., in their research article concluded that the single queue multi server queuing model is better than the multi queue multi server queuing model by using the principle of Mathematical induction. This research article the authors discussed the basic definition of service cost of the each server (Cs) and the waiting cost of the each customer (Cw) and traced the relation between service cost of each server and waiting cost of the each customer, determined the optimal expected service cost and the optimal expected waiting cost and analyzed graphically.

2. Performance Measures of the Multi server queuing Model

The performance measures of $(M/M/S/\infty/FCFS)$ model

The probable number of consumers in waiting line

$$N_q = \left[\frac{1}{(s-1)!} \left(\gamma \big/ \kappa \right)^s \left(\frac{\gamma\kappa}{(s\kappa-\gamma)^2} \right) \right] P_0$$

Where $$P_0 = \left[\sum_{n=0}^{s-1} \frac{1}{n!} \left(\gamma \big/ \kappa \right)^n + \frac{1}{s!} \left(\gamma \big/ \kappa \right)^s \left(\frac{s\kappa}{s\kappa-\gamma} \right) \right]^{-1}$$

The probable number of consumers in the system

$$N_s = N_q + \frac{\gamma}{\kappa}$$

The probable waiting time of a consumer in the waiting line

$$T_q = \frac{N_q}{\gamma}$$

The probable waiting time of a customer in the system

$$T_s = T_q + \frac{1}{\kappa}$$

3. Research contribution

3.1. Definition 1: The expected service cost $(E(S_c))$ is directly proportional to the number of servers in the system.

$E(S_c) \prec S$

$E(S_c) = C_s * S$, here $C_s > 0$ is the service cost per unit time of the each server.

3.2. Definition 2: The expected waiting cost $(E(W_c))$ is directly proportional to the number of customers in the system.

$E(W_c) \prec N_s$

$E(W_c) = C_w * N_s$, here $C_w > 0$ is the waiting cost per unit time of the each customer.

3.3. Expected total cost

The expected total cost of the multi server queuing model is $E(T_c) = C_w * N_s + C_s * S$.

3.4. Lemma 1:

In the multi server queuing model the expected total cost is $E(T_c) = C_w * N_s + C_s * S$.

If $C_s > 0$ then $0 < C_w < \dfrac{E(T_c)}{N_s}$

Proof:

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From the equation $E(T_c) = C_w * N_s + C_s * S$

$$C_s = \frac{E(T_c) - C_w * N_s}{S}$$

Since $C_s > 0$

$$\frac{E(T_c) - C_w * N_s}{S} > 0$$

$$0 < C_w < \frac{E(T_c)}{N_s}$$

3.5. Lemma 2:

In the multi server queuing model the expected total cost is $E(T_c) = C_w * N_s + C_s * S$.

If $C_w > 0$ then $0 < C_s < \frac{E(T_c)}{S}$.

Proof:

From the equation $E(T_c) = C_w * N_s + C_s * S$

$$C_w = \frac{E(T_c) - C_s * S}{N_s}$$

Since $C_w > 0$

$$\frac{E(T_c) - C_s * S}{N_s} > 0$$

$$0 < C_s < \frac{E(T_c)}{S}$$

3.6. Optimality of multi server queuing system

The expected total cost of the multi server queuing model is $E(T_c) = C_w * N_s + C_s * S$.

When $C_s = 0$ then $C_{w0} = \frac{E(T_c)}{N_s}$ and when $C_w = 0$ then $C_{s0} = \frac{E(T_c)}{S}$ and $|C_{s0} - C_{w0}|$ is the absolute difference of C_{s0} and C_{w0}.

For the given total expected cost, the optimization of the multi server queuing model is the corresponding number of servers at which the absolute difference of C_{s0} and C_{w0} is minimum i.e. the optimal point is $\left(S,\ \min\left\{|C_{s0} - C_{w0}|\right\}\right)$.

4. Results and Discussions

In order to determine the optimal service cost per unit time of each server (Cs) and waiting cost per unit time of each customer (Cw), anyone can consider the multi server queuing model with arrival rate (γ), service rate (κ) and the number of servers in the system (S). The performance measures can compute by using TORA software.

Table1. The performance measures of the multi server queuing model (snap shot)

TORA Optimization System, Windows®-version 2.00
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Monday, April 06, 2020 17:35

QUEUEING OUTPUT ANALYSIS

Title: queue
Comparative Analysis

Scenario	c	Lambda	Mu	L'da eff	p0	Ls	Lq	Ws	Wq
1	16	150.00000	10.00000	150.00000	0.00000	25.95114	10.95114	0.17301	0.07301
2	17	150.00000	10.00000	150.00000	0.00000	18.90204	3.90204	0.12601	0.02601
3	18	150.00000	10.00000	150.00000	0.00000	16.80667	1.80667	0.11204	0.01204
4	19	150.00000	10.00000	150.00000	0.00000	15.91582	0.91582	0.10611	0.00611
5	20	150.00000	10.00000	150.00000	0.00000	15.48129	0.48129	0.10321	0.00321
6	21	150.00000	10.00000	150.00000	0.00000	15.25580	0.25580	0.10171	0.00171
7	22	150.00000	10.00000	150.00000	0.00000	15.13566	0.13566	0.10090	0.00090
8	23	150.00000	10.00000	150.00000	0.00000	15.07120	0.07120	0.10047	0.00047
9	24	150.00000	10.00000	150.00000	0.00000	15.03679	0.03679	0.10025	0.00025
10	25	150.00000	10.00000	150.00000	0.00000	15.01865	0.01865	0.10012	0.00012

For the analysis of the definition1 and definition 2 consider the service cost per unit time of the each server (Cs) is $100 and the waiting cost per unit time of the each customer (Cw) is $100. The expected waiting and expected service costs are computed as follows:

Table 2. The expected waiting and expected service costs

S	Ns	E(Sc)	E(Wc)
16	25.95114	1600	2595.114
17	18.90204	1700	1890.204
18	16.80667	1800	1680.667
19	15.91582	1900	1591.582
20	15.48129	2000	1548.129
21	15.2558	2100	1525.58
22	15.13566	2200	1513.566
23	15.0712	2300	1507.12
24	15.03679	2400	1503.679
25	15.01865	2500	1501.865

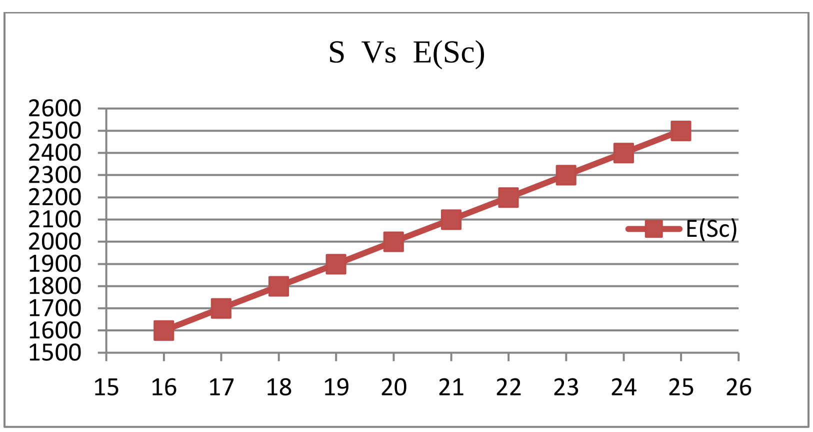

Fig. 1: Number of servers in the system versus expected service cost

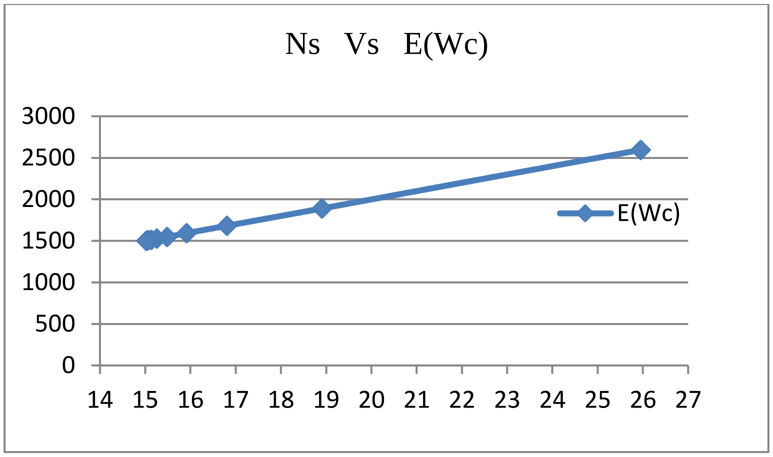

Fig. 2: Number of customers in the system versus expected waiting cost

For the given total expected cost $E(T_c)$ which is $2000 then the optimization of the multi server queuing model computed as follows:

Table 3: Optimization of multi server queuing model for given (Etc) is $2000

S	N_s	$E(T_c)$	C_{s0}	C_{w0}	Absolute Diff.
16	25.95114	2000	125	77.06791	47.9320947
17	18.90204	2000	117.6471	105.8087	11.83837362
18	16.80667	2000	111.1111	119.0004	7.889261955
19	15.91582	2000	105.2632	125.6611	20.39797675
20	15.48129	2000	100	129.1882	29.18820072
21	15.2558	2000	95.2381	131.0977	35.85958564
22	15.13566	2000	90.90909	132.1383	41.22918387
23	15.0712	2000	86.95652	132.7034	45.74691263
24	15.03679	2000	83.33333	133.0071	49.67377789
25	15.01865	2000	80	133.1678	53.16776142

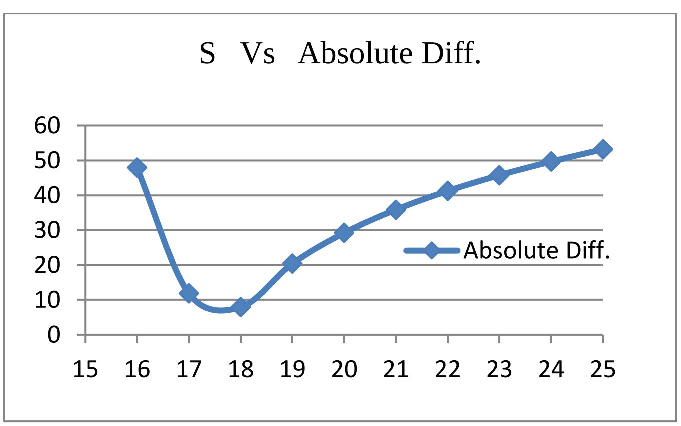

Fig. 3: The number of the servers versus absolute difference of Cs0 and Cw0

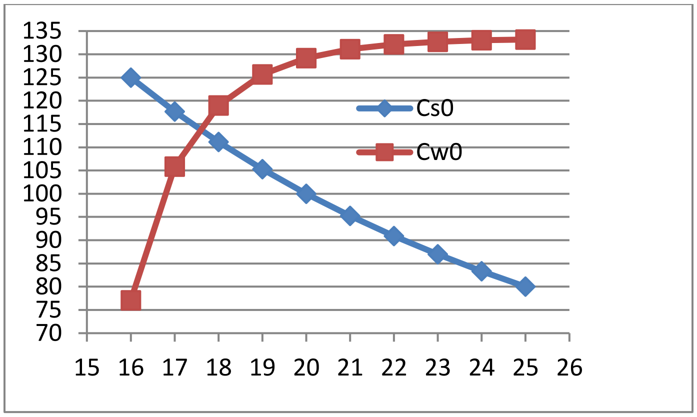

Fig. 4: The number of the servers versus Cs0 and Cw0

5. Conclusion

The conclusions of the research articles as follows:

i. From the fig.1 as the number of servers increased in the system correspondingly the expected service cost is increased which means the expected service cost is directly proportional to the number of servers in the system.
ii. From the fig.2 as the number of customers increased in the system correspondingly the expected waiting cost is increased which means the expected waiting cost is directly proportional to the number of customers in the system.
iii. From the fig. 3 the optimization of multi server queuing model is (18, 7.889), the optimal number of the servers are 18 at which the corresponding absolute difference of Cs0 and Cw0 is minimum.
iv. From the fig.4 the optimal number of servers is 18 where the curve of Cs0 and Cw0 are intersecting.

In this research article the authors discussed the basic definition of the expected service and waiting costs of the multi server queuing model and computed the optimal required number of servers and computed the possible range of the expected service and waiting costs but not find the optimal expected service and waiting costs.

6. References

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Authors

Dr.S.Vijay Prasad has been working as an Assistant professor in the Department of Mathematics, Koneru Lakshmaiah Education Foundation (Deemed to be University), A.P., India since 2 years. He completed his Ph.D. from Vikram University in 2016, Ujjain, M.P., India. He completed his M.Phil. in Mathematics during the academic year 2009-2010, from Vikram University, Ujjain, M.P., India. He completed his post-graduation (M.Sc. Mathematics) during the academic year 2005-2007, from DAVV, Indore, M.P., India. He has published 17 research articles in reputed international journals. His areas of research are Operations Research, Queuing theory. His total experience in teaching is 5 years.

Dr. Ranadheer Donthi has been working as a professor in the Department of Mathematics, St. Martin's Engineering College, Dhulapally, Kompally, Hyderabad, Telangana, India since 1 years. He completed his Ph.D. from Kakatiya University in year 2016, Warangal, Telangana, India. He completed his M.Phil. in Mathematics during the academic year 2009-2010, from Sri Venkateswara University, Tirupathi, A.P., India. He completed his post-graduation (M.Sc. Mathematics) in the year 2006, from Kakatiya University, Warangal, Telangana, India. He has published 14 research articles in reputed international journals. His areas of research are Internet traffic, Operations Research, Queuing theory. His total experience in teaching is 14 years.